

## Functional Analysis HW 2

Deadline: 20 Feb 2017

1. Let  $X$  and  $Y$  be normed spaces and  $T : X \rightarrow Y$  be a linear map. Show that  $T$  is bounded if and only if  $T$  sends every Cauchy sequence in  $X$  to a Cauchy sequence in  $Y$ .
2. Let  $X$  be the set of all  $C^1$ -functions defined on the open interval  $(0,1)$  such that  $x$  and its derivatives  $x'$  both are bounded. Let  $Y$  be the space of all bounded continuous functions on  $(0,1)$ . Suppose that  $X$  and  $Y$  are equipped with the sup-norm. Define  $T : X \rightarrow Y$  by  $Tx = x'$  for  $x \in X$ . Show that  $T$  is an unbounded operator but the kernel of  $T$  is closed.